Counting – a deceptively simple skill

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Many people consider counting to be a simple skill, but that leads us to be deceived – we often hear young children recite a string of words and assume they can count.

Sharing attention to counting

The fact that we might not be focusing on the same things when we count together with children is supported by research into children's beliefs about counting. Research suggests that, despite being able to perform quite well, a high proportion of children do not understand the adult's purposes of counting before they start school (Munn, 1997)[ii].

Research showed that pre-school children had ideas about counting as a social experience, they thought it was for pleasure, or to please others, but it was rare for such young children to explain that it was 'to know how many'. Their impressions were that it was a social or playful activity, but they did not connect it with quantification. Penny Munn could see a combination of things that alongside the pre school child's lack of appreciation of other people's mental activities might contribute to lacking understanding about *why* they were counting with an adult.
She observed:

- during joint counting activities between very young children and adults the process of saying words seems important, but the actual aim of finding quantity is often not emphasised
- the strength of children's own natural concentration on the physical aspects of counting activities, touching and handling, often obscures the intended mental function of finding quantity
- children are dependent on adults to provide for them, they have no urgent real reason to check and tally, so counting is usually a game
- count words often occur as parts of games that are not to do with quantity, e.g. One, two, three – go.

If such typical pre-school children have not yet seen the meaning beyond the social facade of the counting they do with adults. How do children with learning difficulties fare in the process? Is there any wonder they sometimes watch our faces, to check if they are doing the right thing, rather than look at the objects they are counting. If we could read the thoughts behind their eyes what would be the question in their mind. 'Does he want me to touch them? Shall I make that '*Won too three noise?*'

Penny Munn has a number of suggestions that might help us make our scaffolding of early counting and modelling activities more effective:

1. We need to establish *what* children believe we are doing together when we count.
2. Counting has many sub skills and we need to be able to see which of these are the things that they think we want them to do. *e.g. itemising, ordering, naming.*
3. Though we need to be aware when their counting is only a form of recitation, we also need to take their non-numerical counting seriously.
    a. *Though it may not yet be mature and related to quantities their recitation of number words is important practise in establishing word order.*

   b. *This recitation of number words is also an important point of social contact we can share with them and from which we can help them establish connections with quantity.*
4. We need to make the purpose of counting explicit. She suggests that as we are involving them in the use of counting words we could also muse aloud providing feedback about aspects of quantity that are involved. Children respond well to this kind of incidental information, and even as adults we use forms of thinking aloud to direct our actions. (*See private speech in chapter on communication.*)
5. We need to encourage children to use numerical goals, making use of counting for tallying, checking and comparing items and events.

Typical children often have both:
1. a practical knowledge of small quantities, and
2. are able to produce a series counting words.

When adults hear children counting there is a tendency for them to think that they have integrated these two aspects of counting. But this is not always the case, and these connections must be made if children are to *count meaningfully* or to understand the role and purpose of counting in practical calculations.

The process of counting

In order to count a group of objects children must be able to itemise them and tag each with a number name. Through imitating others counting children learn to connect experiences of

- looking at and touching objects

With

- The verbal symbols of the count words.
- Understanding the meaning and consequences of the connections.

So co-ordinating visual and tactile activities with verbal labelling are important parts of counting *but* meaningful counting requires an understanding of the quantities involved and an understanding of the purpose of counting.

The Principles of counting

To count accurately and with meaning there are five principles that must be understood and applied (Gelman and Gallistel, 1978)[ii]

Principles about how to count
- The one to one principle
- The stable order principle
- The cardinal

Principles about what can be counted
- The abstraction principle
- The order irrelevance principle

About the 'How to count principles'

The one to one principle

Understanding and ensuring each item receives one and only one tag, which requires:

- *Tagging. S*ummoning up and applying distinct names one at a time.
- Physically keeping track, or mentally *partitioning*. Which items have already been counted and which remain.

It is necessary to realise that the name tags are specially for counting with, they are nothing to do with other characteristics of the items being counted.

Teaching related to the one to one principle

In the early stages of learning to count children may be vague or imprecise about their pointing, they wave their fingers in the general direction, but often let the rhythm of the oral counting sequence dominate the speed at which they count, and consequently lose correspondence. Teaching needs to make them more aware of the importance of coordinating the itemising and tagging. Promoting strategies that will help them to notice if they have double counted or missed items e.g.

- practising arm gestures and pointing at large objects
- controlling clapping and counting events
- controlling sequential touching
- moving items – to keep track on which have been counted and which are left
- placing counters or other tallies on or next to the objects being counted
- using number lines or tracks as templates to place items on as they are counted
- making marks next to objects being counted
- using the index finger to touch point (d*eclarative point)*
- *using the index finger to point without touching*
- developing eye pointing and nodding.

There are many times when working or playing together we can make sure good itemising or marking takes place. There are many different levels of itemising, vocalising, and naming, all too often in games we simply expect or prompt the child to count successfully when it would be more fruitful to have a less ambitious objective. This is a prime area for breaking skills down; games provide motivating opportunities to practise the parts of counting.

The stable order principle

The name tags must always be used in a stable order.

Using number names to provide ordinal names for things being counted presents the child with the problem of remembering a long list. (Miller, 1956)[iii].
This is the role of the traditional nursery rhyme – developing the Childs ability to reflect upon the sound patterns of words, building up the sequence of number words as sound patterns. They will be learned and extended in short chunks [1] and we need

[1] P scales recognise progression in the extent of a child's rote sequence – and that a differential will exist between the rote sequence and accurate counting.

to be aware of the current extent of a pupil's performance so that in our modelling and during peer interaction we can emphasise the next steps[2].

Teaching related to the stable order principle

We need to recognise that many of our pupils have grown past the age at which nursery rhymes are appropriate – but still need this phonological development and practice. In our teaching we need to recognise the valuable role of the way we use our own voices when we model and teach. We must be aware of the power of intonation and rhythm, in offering connections and prompts that make it easier to create memorable 'chunks' or connections that help learning a sequence of words.

Some suggestions that might form the basis of, or be included in, learning activities are listed below:

- Sound and word play to promote discrimination.
  - *emphasising key sounds within word, identifying word from first sound or sound profile, creating strings of sound.*
- Linguistic rhyme and rhythm games incorporating number words
  - *Raps, chants, catch phrases, modified songs – downloaded video clips e.g. u tube – Sesame St count it higher - Sesame St Feist 1234.*
- Games to highlight the importance of coordinating words with items
  - *including fast counting, slow counting, pauses, rhythmic counting and counting without rhythm*.
- Intonation emphasising important aspects of the sequence
  - *punctuating the sequence well and emphasising the last item counted to emphasise the cardinal number – see below.*
- Using gestures and movement along with sounds, and visual representations e.g. number lines or rows of items or people.
  - *hand and arm gestures, pointing, clapping, stamping, nodding.*

Though the emphasis within the suggestions above is on learning the string of number words we should also bear in mind

- the importance of fingers movements and gestures accompanying speech
- the importance of coordinating the speech and movement with visual arrays such as number lines, rows of objects or people.

Martin Hughes (1986)[iv] notes how children still resort to pointing and tapping to assist their counting even when objects are out of sight and a critical factor for building a number system is connecting the spatial/perceptual representations we have in number sense with learning the count sequence.

The cardinal principle

The final number represents the size of the set.

When the child understands this principle they recognise that earlier numbers were temporary steps towards the last number tag, which is special, because it is the **cardinal number and represents 'how many' items have been counted**.

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[2] The central role of learning from others is highlighted by Vygotsky's 'zone of proximal. development', which indicates how much further a child can go when learning with the support of a teacher, parent, carer or peer.

Appreciating the importance of *cardinality* is an important milestone in a child's mathematical development; it is a keynote in understanding that the process of counting is not just sound making or a game but actually has meaningful and useful purposes – *related to finding out how many.* It is a principle that a child must understand before they can begin to carry out **addition by 'counting on'**.

It is necessary for children to understand this principle before they can use the technique of 'counting on' to carry out addition or to determine or compare the equivalence of two groups.

Fully grasping the cardinal principle depends on understanding the previous two principles, it therefore matures after them.

There are four phases in its development (Fuson and Hall 1983)[v]. Reciting the last number with no clear idea that it relates to quantity – p*robably because they think it is the response the adult expects.*

- Understanding that the last number of the count relates to the quantity. In which case they may be able to understand that they can respond to a request like give me three – by counting.

- Understanding the progressive nature of cardinality i.e. if they are stopped in the middle of a count they can say how many they have counted so far, and then carry on. Being able to compare size represented by numbers, or understand that the next number in a sequence represents a larger quantity
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Teaching related to the cardinal principle

It is useful to observe if the child has an appreciation of the purposes of counting and if so at which level they apply the cardinal principle, so that we can arrange appropriate individual modelling, and also be aware of what to emphasise as we work with them in practical activities or games with peers.

- To understand the last number of the count relates to the quantity.
- When the child's counting is recitation without meaning our teaching should include modelling counting things for a practical purpose. We can emphasise the special importance of the last item in a count by intonation, and use checking to reinforce purposefulness.

- Making and naming groups *e.g. counting the row of objects – and gathering them together as the count finishes – or restating the cardinal as they are gathered.*

- To understand the progressive nature of cardinality.
- Provide models of pausing and continuing in practical counting activities.
- Demonstrate adding one more model and involve pupil in checking result.
- Model and involve pupil in solving practical addition problems by counting on.
- Give pupil opportunities to use apparatus or objects in play and exploration that illustrate progression of number, experiencing staircase image or nesting.

- It may be necessary to be aware of pitching the level of counting within the zone of the child's established number words and accurate perception of groups.

- Being able to compare size represented by numbers.
- Use counting to compare groups that have been named – and associated to a numeral, checking answers by counting again.
- Using templates or number lines to make visual comparisons of groups that have been named.
- Compare photos or pictures of groups – compare and count.
- Model and involve pupil in adding or finding one more to groups and checking.

- Provide models and involve pupils in practical activities of carrying out addition by 'counting on'.

The principles about what can be counted

The abstraction principle

Counting can be applied to any collection – real or imagined.

Adults realise that they may count physical or non-physical entities, similar and dissimilar things, objects that are not present or even ideas. Young children on the other hand count physically present items and they group things in accordance with how they see immediate relationships. Variations in material properties or position may affect their view as to whether an item can or should be included in a count.

When we are working with or assessing pupils at early stages of development, who may have difficulty with abstract thinking we need to be aware of what they conceive as allowable within a counting sequence.

- What they might think about including or leaving out of the count on grounds of physical properties, position and so on.
- Are they able to understand they can count objects they cannot see?
- Can they count events as they happen, and events that occur elsewhere?
- Can they count ideas?

Teaching related to the abstraction principle

- Sorting by different criteria – discussing different reasons for groupings.
- Model counting groups of things that are nor identical.
- Counting items before hiding, or storing in drawers or cupboards, bag, thinking about them and counting them when they are not visible.
- Using photographs or video of items to count them when they are not there.
- Using narratives and remembering events, using fingers tallying, mark making and counting to remember and record things that happened, use photographs or collected objects to check.

- Use narratives and discussion about the future – counting things that might happen or things we think about – e.g. How many of us will come to school tomorrow?

The order irrelevance principle

The order in which items are counted is irrelevant; the same cardinal value will be reached.

This principle requires knowledge about the previous four principles. Grasping it entails understanding that each counted item is still an individual thing, not a 'one' or 'two' etc. because number name tags are temporarily given for the purpose of counting, not renaming things and they do not necessarily adhere to the objects once the counting is finished.

Whatever order the objects are counted in the same cardinal result occurs. It is necessary to grasp this principle in order to be able to generalise the use of counting flexibly as a tool without having to put things in a line. It helps us confirm the consistency of the quantity of a group and it is confidence in that consistency that enables us to be sure about making comparisons. Such confidence helps us to override the messages of perception that may confuse us when spatial changes make things appear bigger, and it may therefore underlie our ability to recognise the conservation of number.

Teaching related to the order irrelevance principle

- Developing pointing strategies
- Counting linear ordered groups in different order and forward and back
- Making linear groups and recounting them
- Rearranging linear groups to random and re-counting
- Counting and rearranging random groups
- Counting groups rearranging spatial distributions
- Discriminating and counting specific items from pictures or photographs of mixed groups.
- Pointing at and counting items that are distributed about the room
- Keeping tally of I spy type games

Some confusion about language when counting

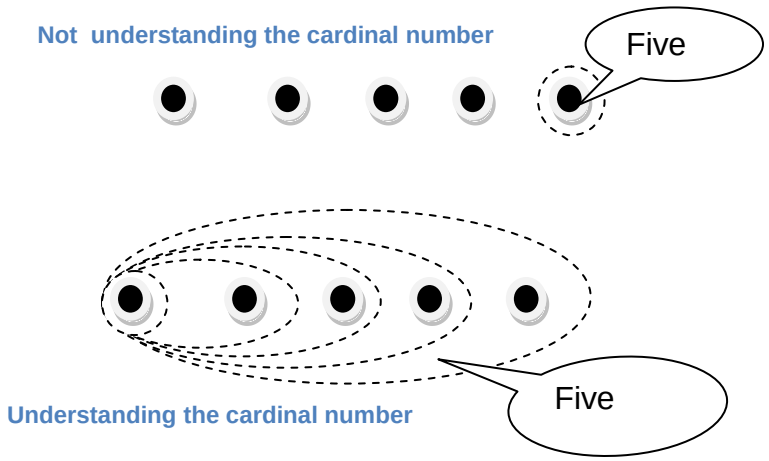
- When we are counting we use the number words in different ways:

1. as **ordinals** – to keep order as we tag each item
2. as a **cardinal** when we name the size of the group.

Children may not always appreciate these distinctions for example:

If we ask a typical four year old to count a row of five objects they may count them correctly and declare that there are 'five'. But If we then ask them – 'so show me five' they will often point to the fifth object – saying 'that one'.

- This suggests that the child is interpreting the words one, two, three etc. as names for the individual elements in the count *like Monday, Tuesday, Wednesday are names of the sequence of elements of the week*. So for that child on that occasion the word '*five'* stands for the last object in the group – and not for the entire group. See Fig 1 below.



The lower diagram also illustrates 'hierarchical inclusion' - the idea that number is progressive and bigger numbers contain all the smaller numbers before them

Fig 1

To carry out the process of counting and naming the child must understand

1. sometimes we use number words temporarily as 'ordinals '– to keep track of the order
2. then we can use them as 'cardinals' – to name the size of the group.

   In order to understand that you can describe the 'quantity' in the group you have to appreciate that a larger number includes all the smaller numbers. This is another of the 'big ideas' – *hierarchical inclusion.*[3]

When we are using or modelling the use of number words with children for practical purposes it is important that we try to make it clear when we are using them as tags to show or keep order – and when we are using the as cardinals to name the quantity.

Just to add a little more confusion sometimes number names are used just as names (nominals) *e.g. the number 4 bus.*

More big ideas

When children have become fluent in applying the principles of counting there are still a number of 'big ideas' – concepts – they need to understand to for their counting to be a useful part of their mathematical thinking. These include

[3] The development of the cardinal principal and understanding hierarchical inclusion are interrelated – each is helped by the other.

- Hierarchical inclusion - *understanding that numbers build by exactly one each time they progress – and that they nest within each other by this amount.*
- Compensation – which entails:
    - Appreciating the balance of increase and decrease.
    - Applying understanding of whole and part relations within numbers.
    - Understanding that different combinations can make up the same number.

Readers interested in the big ideas will find them discussed in more detail in The Equals Guide to Mathematics[vi] and in 'Young mathematicians at work' Fosnot and Dolk 2001[vii]

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[i] Munn P. (1997) Childrens beliefs about counting. In Thompson I. (ed) *Teaching and learning early numbe. Pgs 9-19 .* Buckingham , Philadelphia. Open University Press.

[ii] Gelman,R. Gallistel, R.C. (1978) The Childs Understanding of Number. Cambridge. USA. Harvard.U.P.

[iii] Miller, G.A. (1956) The Magical number 7 + or – 2 Psychological Review, 63, 81-97.

[iv] Hughes M. (1986) Children and Number. Oxford. Blackwell.

[v] Fuson. KC & Hall JW (1983) The acquisition of early number word meanings. A conceptual analysis and review. In Ginsburg, H.P. (ed) The development of mathematical thinking. NY Academic Press.

[vi] Staves (2009) The EQUALS guide to mathematics – supporting access for pupils working towards the national curriculum. – Section 4 Chapter 6 pgs 18 -20 . EQUALS Tyne and wear UK www.equals.co.uk

[vii] Fosnot CT. Dolk M. 2001 . Young mathematicians at work .Heinemann. Portsmouth NH USA